Proposals for Inclusion of the Electrode Radius in Grounding Systems Analysis Using Interpolating Element-Free Galerkin Method

Ursula C. Resende, Rafael S. Alípio and Maisa L. F. Oliveira

Electrical Engineering Department, Federal Center for Technological Education of Minas Gerais, MG 35510-000, Brazil, resendeursula@des.cefetmg.br, rafael.alipio@gmail.com and maisa.laila@gmail.com

In this work the Interpolating Element-Free Galerkin meshless method is presented as a new, simple and accurate technique to investigate electrostatic grounding problems. Two proposals for inclusion of the electrode radius in the analysis are proposed. The problem of a vertical electrode introduced in a homogeneous soil is considered as a study case. The results of grounding resistance obtained from different values of radius are compared with those generated by Method of Moments.

***Index Terms*—Element-Free Galerkin Method, grounding problems, meshless method, vertical grounding electrodes.**

I. INTRODUCTION

VERTICAL grounding electrodes are one of the simplest and most commonly used means of earth termination of electrical systems. Their function is essentially to provide a low-resistance path for the flow of fault currents towards the soil and to ensure a smooth distribution of the Electrical Potentials (EP) developed on the ground surface.

Simplified approaches can be used to determine the Grounding Resistance (R_T) and EP. However the proper calculation for practical problems should be done by employing numerical techniques. Using integral equation methods, such as the Method of Moments (MoM), this kind of problem can be successfully evaluated [1]. However, for non-homogeneous soils the method becomes computationally intensive. On the other hand, complex structures and inhomogeneities are well treated by differential equation methods, such as the Finite Element Method (FEM) [2]. However, the process for mesh generation in FEM requires considerable computational effort when the problem involves complex geometries. In the recent years a new class of numerical method, Meshless Method (MM), has been developed for the solution of Partial Diferential Equations. MM does not require a mesh structure and the solution is obtained using only a cloud of nodes spread throughout the region of interest. This feature makes MM appropriate to deal with complex geometries and inhomogeneities. Among the MM available in the literature, the Interpolating Element-Free Galerkin Method (IEFGM) is one of the most investigated and used because of its simplicity and accuracy [3]-[5].

An important feature of the numerical methods applied to grounding problems is how the electrode radius is considered [6]. The correct modeling of this parameter is essential to ensure the results accuracy. In this work the IEFGM is used to study an electrostatic grounding problem and two methods are proposed in order to accurately consider the electrode radius.

II. GROUNDING PROBLEM MODELING

A thin copper electrode, with length Le and radius Re, introduced in a homogeneous soil with conductivity σ is the problem under analysis. The soil geometry is a semi hemisphere with radio Rh=10Le. The electrode is introduced into the top of the hemisphere in $\rho = z = 0$, with a constant electric potential Ve. The Laplace equation, $\nabla^2 V=0$, can be used to solve the problem and since it is axisymmetric ($\partial V/\partial\phi=0$) only the ρz plane needs be considered. Therefore, the domain of the study Ω is 2D, as illustrated in Fig. 1, and the associated Laplace equation is [7]:

$$\frac{1}{\rho}\frac{\partial V}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right)+\frac{\partial^2 V}{\partial^2 z}=0\,, \tag{1}$$

where $V(\rho, z)$ is the electric potential. $\partial V/\partial \boldsymbol{n} = 0$ on Γ_n and $V = V_d$ on Γ_d are the Neumann and Dirichlet boundary conditions, respectively, and $\hat{\boldsymbol{n}}$ is the outward normal unit vector. The weak form of the problem, $\int_\Omega \nabla w\cdot\nabla V d\Omega=0$, is obtained using the Method of Weighted Residuals and test function w [2].

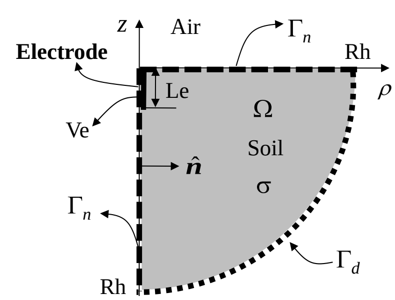

Fig. 1.Grounding problem.

III. IEFGM FORMULATION

For the Mehsless Method approach a set of N nodes is placed in Ω. Each node, I, is a point $\mathbf{x}_I(\rho, z)\in\Omega$ for which a shape function, $\Phi_I(\mathbf{x})$, is associated. $\Phi_I(\mathbf{x})=0$ over the whole domain, except near the corresponding node. Therefore, the unknown function can be approximated by the following equation [8]:

$$V(\mathbf{x})\approx V^h(\mathbf{x})=\sum_{I=1}^{N}\Phi_I(\mathbf{x})\upsilon_I\,, \tag{2}$$

where $\mathbf{x}=(\rho, z)$ and υ_I is the unknown coefficient of node I.

Using Galerkin method the linear system [$\boldsymbol{K}$][υ]=0 is obtained, with $K_{ij} = \int_\Omega \nabla\Phi_i \cdot \nabla\Phi_j \, d\Omega$.

In MLS approximation the coefficients υ are determined by minimizing a weighted discrete L^2 norm, for which the singular weight function is $W(r_I) = 1/(r_I^n + \beta^n)$, where β is a constant small enough to ensure no division by zero, n is a constant adjusted to improve the results accuracy, $r_I = |\mathbf{x} - \mathbf{x}_I| / d_I$, $d_I = \alpha\, d_c$ is the support of the weight function, α is a scaling factor for the influence domain and d_c is the nodal distance [5].

IV. Electrode Radius Modeling

The proper inclusion of the electrode radius is a fundamental aspect to ensure the accuracy of the results in the numerical analysis of grounding problems. In this work, two techniques are proposed in order to consider this parameter using IEFGM. For the first proposal, the Real Radius Approach (RRA), the distribution of nodes is carried out as illustrated in Fig. 2 and α=1.5 for all nodes in Ω. However, for thin electrodes the RRA becomes computationally intensive. Then, an alternative approach is presented to include the electrode radius effect, Equivalent Radius Approach (ERA). For this approach the distribution of nodes is made as illustrated in Fig. 3, α=1.16 for nodes representing the electrode and α=1.5 for remaining nodes. So, the radius effect is introduced by adjusting the influence domain size of the nodes representing the electrode.

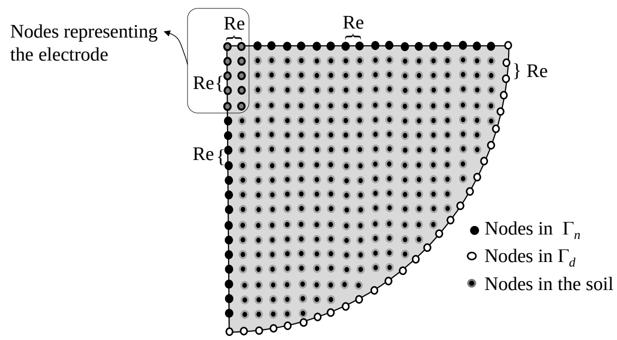

Fig. 2. Nodes distribution RRA.

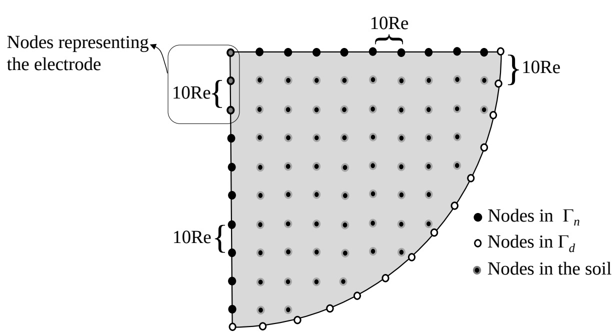

Fig. 3. Nodes distribution ERA.

V. Numerical Results

Over an equipotential γ it is possible to calculate the electric field, $\mathbf{E}$=$-\nabla V$ and the electric current dispersed to the earth:

$$I_T = \int_0^{2\pi} \int_\gamma \sigma \mathbf{E}\, r^2 \sin(\theta)\, d\gamma\, d\phi \,, \tag{3}$$

where r, ϕ, θ correspond to the spherical coordinates of the points over the equipotential. The grounding resistance is obtained using Ohm's law, R_T=Ve/I_T.

In this work, an electrode with Le=1 m and Ve=1 V and a fictitious homogeneous soil with σ=1 S/m was chosen in order to show the effectiveness of proposed approaches. The values of V_d were calculated using an analytical process. The values of I_T and R_T for different values of Re are presented in Table I and Table II, together with the results obtained using MoM. As it can be verified, both results are very close, which demonstrates the IEFGM accuracy. In the full paper, the analysis of inhomogeneous soils will be performed.

TABLE I
I_T AND R_T – ENCAPSULATED ELECTRODES

Re (m)	NN[1]	IEFGM (RRA)		MoM
		I_T(A)	$R_T(\Omega)$	$R_T(\Omega)$
0,04	49723	1,833	0,546	0,551
0,05	31920	1,960	0,510	0,513
0,06	22245	2,063	0,485	0,483
0,07	16388	2,184	0,458	0,457
0,08	12587	2,289	0,437	0,434
0,09	9987	2,386	0,419	0,414
0,1	8114	2,484	0,403	0,397

1 - Number of nodes used in the analysis

TABLE II
I_T AND R_T – CONVENTIONAL ELECTRODES

Re (m)	NN[1]	IEFGM (ERA)		MoM
		I_T(A)	$R_T(\Omega)$	$R_T(\Omega)$
0,00635	19885	1,172	0,853	0,856
0,00794	12786	1,224	0,817	0,820
0,00953	8915	1,255	0,797	0,790
0,0127	5069	1,369	0,730	0,742

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References

[1] Dawalibi, D. Mukhedkar, "Parametric analysis of grounding grids," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-98, no. 5, pg. 1659–1668, setembro/outubro de 1979.

[2] A. F. Peterson, S. L. Ray, and R. Mittra, *Computational Methods for Electromagnetics*, Piscataway, Wiley-IEEE Press, 1997.

[3] T. Belytschko, Y. Y. Lu, and L. Gu, "Element-free Galerkin methods," *International Journal for Numerical Methods in Engineering*, vol. 37, pp 229-256, 1994.

[4] G. N. Marques, J. M. Machado, S. L. Verardi, S. Stephan, and A. J. Preto, "Interpolating EFGM for computing continuous and discontinuous electromagnetic fields," *Intern. Journal for Computation and Mathematics in Electrical and Electronic Engineering*, pp. 1411–1438, 2007.

[5] U. C. Resende, E. H. R. Coppoli, and M. M. Afonso, "A novel approach for electromagnetic scattering using EFG interloping moving least square method," *IEEE Trans. on Magnetics*, vol. 51, nr. 3, 2015.

[6] T. Noda e S. Yokoyama, "Thin wire representation in finite difference time domain surge simulation," *IEEE Transactions on power delivery. Vol. 17. No 13*, pp. 840-847, July 2002.

[7] U. C. Resende, R. S Alípio, M. L. Oliveira and S. T. M. Gonçalves, "Analysis of grounding problems using EFG-IMLS meshless method," ISEF2015, Valence, Spain, 2015.

[8] G. R. Liu, *MeshFree Methods – Moving Beyond the Finite Element Method*, CRC Press, Boca Raton, 2009.